# Math 103 Day 9: Related Rates 

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## Outline

## (1) Related Rates

Related Rates is the most important application of calculus we have seen so far.

Example Air is being pumped into a spherical balloon so that its volume increases at a rate of $10 \frac{\mathrm{~cm}}{\mathrm{~s}}$. How fast is the radius of the balloon increasing when the diameter is 4 cm ?

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## How To Approach These Problems

(1) What quantities are given in the problem?
(2) What is the unknown?
(3) Draw a picture of the situation with labels.
(9) Write an equation that relates the quantities.
(5) Finish solving the problem.

Example A water tank has the shape of an inverted circular cone with base radius 2 meters and a height of 3 meters. If the water is being pumped into the tank at a rate of $3 \frac{\mathrm{~m}^{3}}{\mathrm{~min}}$, find the rate at which the water level is rising when the water is 2 meters deep.

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ExampleA ladder 6 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of $.5 \frac{\mathrm{ft}}{\mathrm{sec}}$, how fast is the top of the ladder sliding when it is 1 ft above the ground?

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Example A round oil slick uniformly 0.1 cm thick is being fed by a leak in an off shore oil rig at a rate of $2 \frac{\mathrm{~m}^{3}}{\mathrm{sec}}$. Sea turtles have bad eyesight and only see the oil as it is nearly on top of them. If sea turtles swim at a rate of $1 \frac{m}{\text { sec }}$ and begins swimming away from the slick as they see it approaching, how far away from the oil rig does a turtle need to be to avoid being overcome by the slick.

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